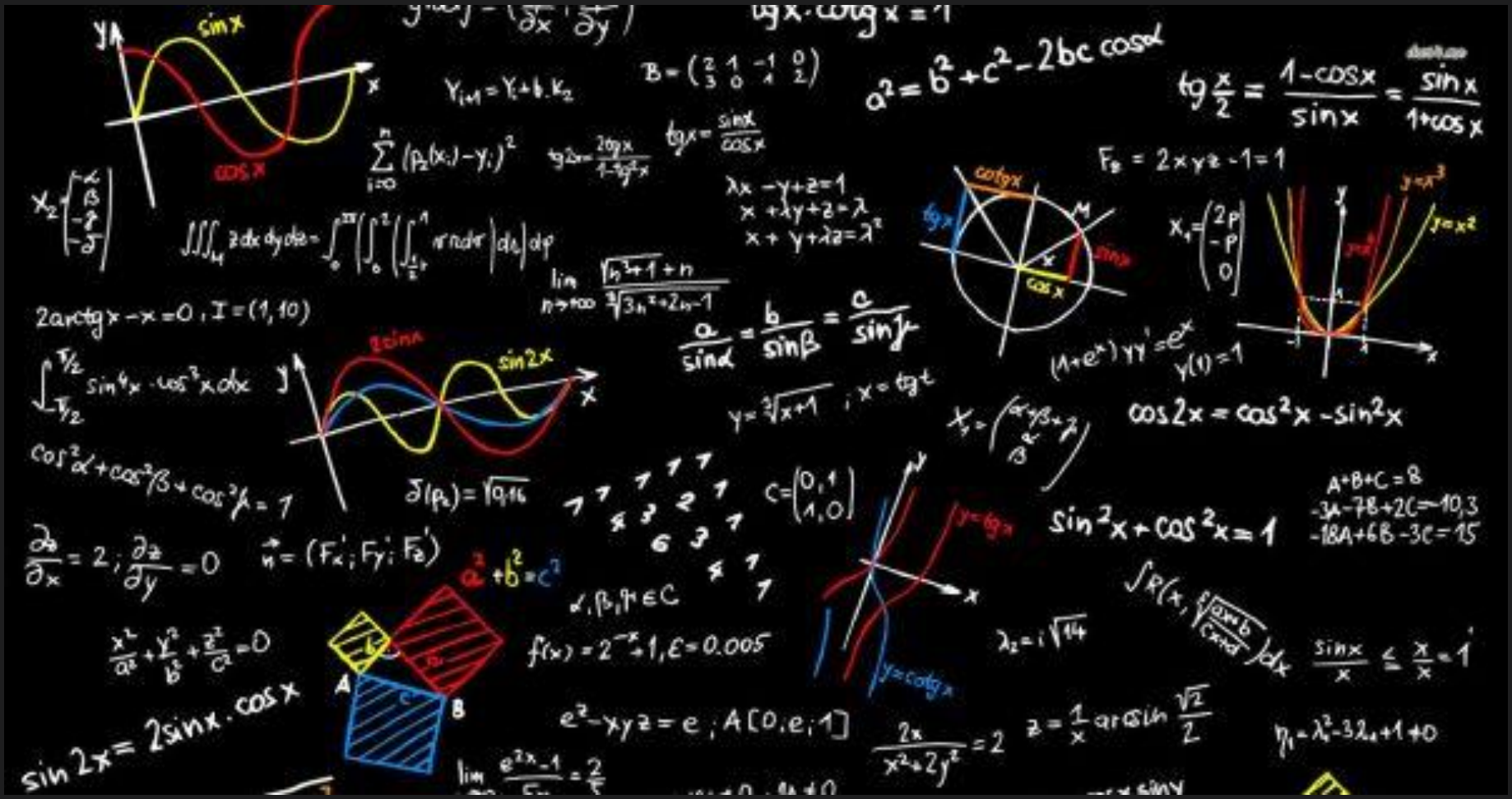
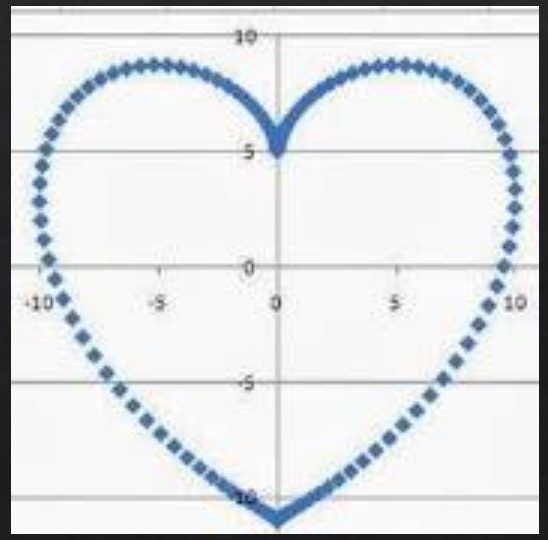


A-Level Maths Induction Revision Material

For each topic there is a help slide with a few examples, a question slide and then an answer slide. Please ensure you are in Slideshow Mode so you can hear the explanations. You should attempt all the questions and then check your answers. If any are incorrect please research the topic until you are confident with it. YouTube is a great resource to use to find help with your mathematics, the links for two useful playlists are linked below.

Exam Solutions

maths made easy



This is the calculator that you must buy for the course

CASIO fx-991EX

≈ £20

YOU MUST NOT USE A CALCULATOR for any questions and all working out will be expected to be seen.

Please bring your solutions and ALL your working out to your first maths lesson.

This will help you prepare for your initial assessment which will show us your level of suitability for the course.

Examples and Questions

INDICES EXAMPLES

Simplify:

* $4x^3 \times 6x^{\frac{1}{2}}$

$= 24x^{(3+\frac{1}{2})}$

$= \underline{24x^{\frac{7}{2}}}$

* $\dfrac{7p^6}{(4p^3)^2}$

$= \dfrac{7p^6}{16p^6}$

$= \dfrac{7}{16}p^0$

$= \dfrac{7}{16}$

INDICES EXAMPLES

Simplify:

$$\frac{10y^{2} + 5y^{-4}}{15y^{-1}}$$

$$= \frac{10y^{2}}{15y^{-1}} + \frac{5y^{-4}}{15y^{-1}}$$

$$= \frac{2}{3}y^{(2--1)} + \frac{1}{3}y^{(-4--1)}$$

$$= \frac{2}{3}y^{3} + \frac{1}{3}y^{-3}$$

INDICES EXAMPLES

Write in index form

* $\frac{1}{g^2}$
* $\sqrt[3]{m}$
* $\frac{2}{\sqrt{p}}$
* $\frac{1}{3w^3}$
* $\sqrt{b^3}$

g^{-2}

$m^{\frac{1}{3}}$

$2p^{-\frac{1}{2}}$

$\frac{1}{3}w^{-3}$

$(b^3)^{\frac{1}{2}} = b^{\frac{3}{2}}$

INDICES QUESTIONS

Simplify:

1) $x^5 \times x^{-2}$

2) $\frac{w^5}{w^{-3}}$

3) $\frac{3}{(3a)^2}$

4) $(mn^2)^5$

5) $2y^2 \times 3y^{\frac{1}{2}}$

6) $12x^4 \div 3x^{-2}$

7) $y^2(y^3 - y^{-\frac{1}{2}})$

8) $\frac{w^2 \times w^{-3}}{w^{-4}}$

Write in index form:

1) $\sqrt{b}$

2) $\frac{1}{g}$

3) $\sqrt{p^3}$

4) $\sqrt[3]{k^2}$

5) $\frac{1}{\sqrt{c}}$

6) $\frac{6}{x^3}$

LINEAR EQUATIONS EXAMPLES

Solve the following:

$x \quad -2x + 7 = 5$

$-2x = -2$

$x = 1$

$x \quad 3x - (x - 2) = 6x - 10$

$3x - x + 2 = 6x - 10$

$2x + 2 = 6x - 10$

$2 = 4x - 10$

$12 = 4x$

$x = 3$

LINEAR EQUATIONS QUESTIONS

Solve the following:

1) $5 - \frac{x}{9} = -1$

2) $2(3x - 1) + 3 = 21$

3) $3x - 2(6x - 3) = 42$

4) $2x - 8 = 6x + 4$

FACTORISING EXAMPLES

Factorise the following:

* $9x^2 - 16$ $= (3x + 4)(3x - 4)$

* $x^2 + 5x + 4$ $= (x + 1)(x + 4)$

4 1
-4 -1

* $18x^3y^2 - 12xy^3$ $= 6xy^2(3x^2 - 2y)$

FACTORISING EXAMPLES

Factorise the following:

* $3x^2 - 5x + 2$

$3 \quad 1$

$2 \quad 1$

$-2 \quad -1$

$= (3x - 2)(x - 1)$

* $2y^2x^2 + x^3 + 2y^3 + xy$

$= (2y^2 + x)(x^2 + y)$

FACTORISING QUESTIONS

Factorise the following:

1. $4x^2 - 9$
2. $x^2 + 8x + 15$
3. $6x^2 - x - 1$
4. $1 - x^2$
5. $15a^2b + 9ab^2$
6. $x^2y^2 - 2xy + 1$
7. $2x^2 - 6xy$
8. $9x^2 - 6x + 1$
9. $3 + 2x - x^2$
10. $25x^2 - 16$
11. $9x^2 + 30x + 25$
12. $6x^2 + 11xy + 4y^2$
13. $7x^2 - 5x - 150$

SURD EXAMPLES

Simplify:

* $\sqrt{72}$

$= \sqrt{36 \times 2}$

$= \sqrt{36} \times \sqrt{2}$

$= \underline{6\sqrt{2}}$

* $\sqrt{75} - \sqrt{27}$

$= \sqrt{25 \times 3} - \sqrt{9 \times 3}$

$= 5\sqrt{3} - 3\sqrt{3} = \underline{2\sqrt{3}}$

SURD EXAMPLES

Simplify:

$$(8 - 2\sqrt{7})^2$$

$$= (8 - 2\sqrt{7})(8 - 2\sqrt{7})$$

$\times$	8	$-2\sqrt{7}$
8	64	$-16\sqrt{7}$
$-2\sqrt{7}$	$-16\sqrt{7}$	$4(7)$

$$= 64 - 16\sqrt{7} - 16\sqrt{7} + 28$$

$$= \underline{92 - 32\sqrt{7}}$$

SURD EXAMPLES

Rationalise the denominator:

$$\cancel{8} \quad \frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8}{3}\sqrt{3}$$

$$\cancel{8} \quad \frac{2-\sqrt{2}}{(1+\sqrt{5})} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{2-\sqrt{2}-2\sqrt{5}+\sqrt{10}}{1+\cancel{\sqrt{5}}-\cancel{\sqrt{5}}-5}$$

$$= \frac{2-\sqrt{2}-2\sqrt{5}+\sqrt{10}}{-4}$$

	2	$-\sqrt{2}$
1	2	$-\sqrt{2}$
$-\sqrt{5}$	$-2\sqrt{5}$	$\sqrt{10}$

	1	$+\sqrt{5}$
1	1	$\sqrt{5}$
$-\sqrt{5}$	$-\sqrt{5}$	-5

SURD QUESTIONS

Simplify:

1) $\sqrt{96}$

2) $\sqrt{90} + \sqrt{250}$

3) $\sqrt{250} - \sqrt{160}$

4) $\sqrt{2} \times \sqrt{14}$

5) $\sqrt{50} \div \sqrt{10}$

6) $(2 + \sqrt{10})^2$

7) $(4 - \sqrt{5})^2$

Rationalise the denominator

1) $\frac{5}{\sqrt{10}}$

2) $\frac{2}{(1-\sqrt{3})}$

3) $\frac{1-\sqrt{5}}{(2+\sqrt{5})}$

COMPLETING THE SQUARE EXAMPLES

Write the following in completed square form:

* $x^2 - 4x + 7$

$= (x - 2)^2 - 4 + 7$

$= (x - 2)^2 + 3$

* $5x^2 - 8x + 3$

$= 5\left[x^2 - \frac{8}{5}x\right] + 3$

$= 5\left[\left(x - \frac{4}{5}\right)^2 - \frac{16}{25}\right] + 3$

$= 5\left(x - \frac{4}{5}\right)^2 - \frac{80}{25} + 3$

$= 5\left(x - \frac{4}{5}\right)^2 - \frac{16}{5} + \frac{15}{5}$

$= 5\left(x - \frac{4}{5}\right)^2 - \frac{1}{5}$

COMPLETING THE SQUARE QUESTIONS

Write the following in completed square form:

1) $x^2 - 2x + 8$

2) $x^2 + 7x - 2$

3) $2x^2 + 4x - 1$

4) $3x^2 - 2x + 6$

SOLVING QUADRATICS EXAMPLES

Solve by factorising:

$$3p^2 - 14p - 5 = 0$$

-5	1
5	-1

$$(3p + 1)(p - 5) = 0$$

$$3p + 1 = 0 \qquad p - 5 = 0$$

$$3p = -1 \qquad p = 5$$

$$p = -\frac{1}{3}$$

SOLVING QUADRATICS EXAMPLES

Solve by completing the square:

$$m^2 + 6m - 1 = 0$$

$$(m+3)^2 - 9 - 1 = 0$$

$$(m+3)^2 = 10$$

$$m + 3 = \pm\sqrt{10}$$

$$m = -3 \pm \sqrt{10}$$

SOLVING QUADRATICS EXAMPLES

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve using the quadratic formula

$$4x^2 - 10x + 5 = 0$$

$a = 4$
$b = -10$
$c = 5$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(5)}}{2(4)}$$

$$= \frac{10 \pm \sqrt{100 - 80}}{8}$$

$$= \frac{10 \pm \sqrt{20}}{8} = \frac{10 \pm \sqrt{4 \times 5}}{8}$$

$$= \frac{10 \pm 2\sqrt{5}}{8} = \frac{5 \pm \sqrt{5}}{4}$$

SOLVING QUADRATICS QUESTIONS

Solve by factorising:

1) $f^2 - 5f = 0$

2) $p^2 + 8p + 16 = 0$

3) $b^2 - 9 = 0$

4) $2x^2 - x - 3 = 0$

Solve by completing the square (leave your answer in simplified surd form):

1) $c^2 + 4c = 0$

2) $z^2 - 10z - 35 = 0$

3) $2y^2 + 12y - 10 = 0$

Solve using the quadratic formula (leave your answer in simplified surd form):

1) $x^2 + 5x + 2 = 0$

2) $2x^2 - 6x - 18 = 0$

SIMULTANEOUS EQUATIONS EXAMPLES

Solve the pair of simultaneous equations

$$4t + b = 11 \quad (1)$$
$$t + 3b = 0 \quad (2)$$

$(1) \times 3 \Rightarrow$

$$12t + 3b = 33$$
$$t + 3b = 0 \quad -$$

$$11t = 33$$
$$\underline{t = 3}$$

$$4(3) + b = 11$$
$$12 + b = 11$$
$$\underline{b = -1}$$

check

$$3 + 3(-1) = 0$$

✓

SIMULTANEOUS EQUATIONS EXAMPLES

Solve the pair of simultaneous equations

$$* \quad 2x + y = 4$$
$$y^2 - 2x^2 = 2$$

$$y = 4 - 2x$$
$$(4 - 2x)^2 - 2x^2 = 2$$

$$16 - 8x - 8x + 4x^2 - 2x^2 = 2$$
$$2x^2 - 16x + 14 = 0 \quad (\div 2)$$
$$x^2 - 8x + 7 = 0$$

$$(x - 7)(x - 1) = 0$$
$$x = 7, 1$$

$$y = 4 - 2(7) = -10$$
$$y = 4 - 2(1) = 2.$$

check $y^2 - 2x^2 = 2$

$$(-10)^2 - 2(7)^2 = 2$$
$$100 - 2(49) = 2$$
$$100 - 98 = 2 \checkmark$$

$$(2)^2 - 2(1)^2 = 2$$
$$4 - 2 = 2 \checkmark$$

SIMULTANEOUS EQUATIONS QUESTIONS

Solve the pair of simultaneous equations:

1)

$$5a + 3b = 9$$
$$2a - 3b = 12$$

2) 4 apples and 1 banana cost £1.70. 2 apples and 1 banana cost 90p. Work out how much one apple costs and how much one banana costs each?

3)

$$3x + 7y = 26$$
$$4x + 5y = 13$$

4)

$$x^2 + y^2 = 25$$
$$y = x - 7$$

5)

$$x^2 - 2x - 25 = y$$
$$y = x - 7$$

ALGEBRAIC FRACTIONS EXAMPLES

Simplify:

$$\frac{y^2 - 3y - 4}{y+1}$$

$$= \frac{(y-4)\cancel{(y+1)}}{\cancel{y+1}}$$

$$= y - 4$$

$$\frac{10}{p+2} \times \frac{p^2 + 3p + 2}{5}$$

$$= \frac{10}{p+2} \times \frac{(p+2)(p+1)}{5}$$

$$= \frac{10(p+2)(p+1)}{5(p+2)}$$

$$= \underline{2(p+1)}$$

ALGEBRAIC FRACTIONS EXAMPLES

Simplify:

$$\frac{3}{p+4} - \frac{5}{p+2}$$

$$= \frac{3(p+2)}{(p+4)(p+2)} - \frac{5(p+4)}{(p+2)(p+4)}$$

$$= \frac{3(p+2) - 5(p+4)}{(p+4)(p+2)}$$

$$= \frac{3p+6-5p-20}{(p+4)(p+2)}$$

$$= \frac{-2p-14}{(p+4)(p+2)}$$

$$= \frac{-2(p+7)}{(p+4)(p+2)}$$

ALGEBRAIC FRACTIONS QUESTIONS

Simplify:

1) $\frac{u^2-7u+12}{u-3}$

2) $\frac{y^2-16}{y^2+4y}$

3) $\frac{4}{v}+\frac{3}{v+1}$

4) $\frac{2}{2f-1}-\frac{5}{2f+1}$

5) $\frac{s+1}{s+2}-\frac{s-4}{s-3}$

6) $\frac{9}{n^2-4n+3}\times\frac{n-3}{2}$

7) $\frac{7}{d-2}\div\frac{8}{d^2-d-2}$

8) $\frac{4r}{r-2}\times\frac{r^2+2r-8}{7}$

STRAIGHT LINES EXAMPLES

* Find the gradient of the line segment between (3, −1) and (8, 9)

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9 - -1}{8 - 3}$$

$$= \frac{10}{5} = \underline{2}$$

STRAIGHT LINES EXAMPLES

* Now find the midpoint of these two points. $(3, -1)$ and $(8, 9)$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

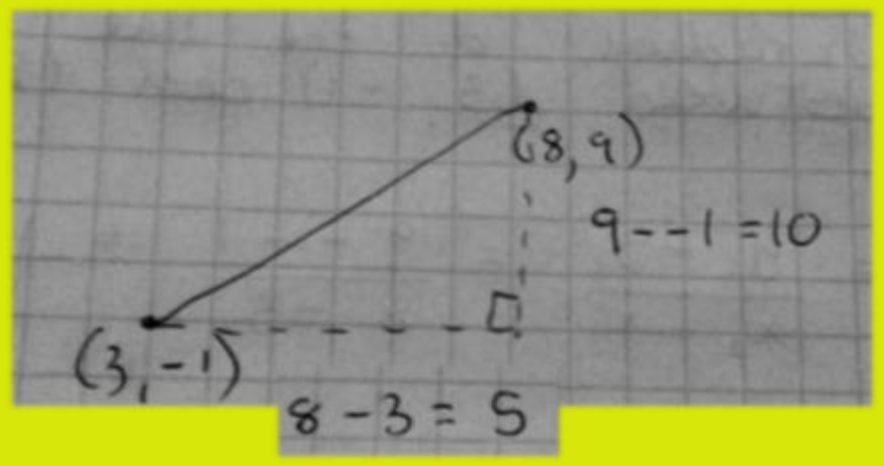
$$= \left(\frac{3+8}{2}, \frac{-1+9}{2}\right)$$

$$= \left(\frac{11}{2}, \frac{8}{2}\right)$$

$$= \left(\frac{11}{2}, 4\right)$$

STRAIGHT LINES EXAMPLES

* Now find the length of this line segment.



So length $= \sqrt{10^2 + 5^2}$

$= \sqrt{125}$

$= \sqrt{25 \times 5}$

$= 5\sqrt{5}$

STRAIGHT LINES EXAMPLES

Find the gradient of this line.

$$3y - 6x + 2 = 0$$

$$3y = 6x - 2$$

$$y = 2x - \frac{2}{3}$$

gradient = 2

STRAIGHT LINES QUESTIONS

Work out the following:

1) A line segment is drawn between $(4,8)$ and $(8,5)$. Find its gradient.

2) A line segment is drawn between $(2,8)$ and $(-6,2)$. Find its midpoint.

3) A line segment is drawn between $(0,-5)$ and $(12,1)$. Find its length (leave your answer as a simplified surd).

Find the gradient of the following straight lines:

1) $y = 3x + 9$

2) $4y = -3x - 2$

3) $y + 3x = 7$

4) $6x - 2y + 17 = 0$

CIRCLE EXAMPLES

Find the centre and radius of the circle with the equation $x^2+y^2=100$

Centre $\underline{(0,0)}$

radius $= \sqrt{100} = \underline{10}$

Find the equation of the circle whose centre is $(0,0)$ and has radius 7

$x^2+y^2=7^2$

$\underline{x^2+y^2=49}$

CIRCLE QUESTIONS

Work out the following:

1) The centre and radius of the circle with equation $x^2 + y^2 = 25$

2) The equation of the circle with centre $(0,0)$ and radius 4.

3) The equation of the circle with centre $(0,0)$ and radius $\sqrt{2}$.

Answers

INDICES ANSWERS

1) x^3

2) w^8

3) $\frac{1}{3a^2}$

4) m^5n^{10}

5) $6y^{\frac{5}{2}}$

6) $4x^6$

7) $y^5 - y^{\frac{3}{2}}$

8) w^3

1) $b^{\frac{1}{2}}$

2) g^{-1}

3) $p^{\frac{3}{2}}$

4) $k^{\frac{2}{3}}$

5) $c^{-\frac{1}{2}}$

6) $6x^{-3}$

LINEAR EQUATIONS ANSWERS

1) $x = 54$
2) $x = \frac{10}{3}$
3) $x = -4$
4) $x = -3$

FACTORISING ANSWERS

1) $(2x+3)(2x-3)$
2) $(x+5)(x+3)$
3) $(3x+1)(2x-1)$
4) $(1+x)(1-x)$
5) $3ab(5a+3b)$
6) $(xy-1)(xy-1) = (xy-1)^2$
7) $2x(x-3y)$
8) $(3x-1)(3x-1) = (3x-1)^2$
9) $(3-x)(1+x)$
10) $(5x-4)(5x+4)$
11) $(3x+5)(3x+5) = (3x+5)^2$
12) $(3x+4y)(2x+y)$
13) $(7x+30)(x-5)$

SURD ANSWERS

1) $4\sqrt{6}$
2) $8\sqrt{10}$
3) $\sqrt{10}$
4) $2\sqrt{7}$
5) $\sqrt{5}$
6) $14 + 4\sqrt{10}$
7) $21 - 8\sqrt{5}$

1) $\frac{\sqrt{10}}{2}$

2) $-1 - \sqrt{3}$

3) $-7 + 3\sqrt{5}$

COMPLETING THE SQUARE ANSWERS

1) $(x-1)^2+7$

2) $(x+\frac{7}{2})^2-\frac{57}{4}$

3) $2(x+1)^2-3$

4) $3(x-\frac{1}{3})^2+\frac{17}{3}$

SOLVING QUADRATICS ANSWERS

1) $f = 0, f = 5$
2) $p = -4$
3) $b = 3, b = -3$
4) $x = \frac{3}{2}, x = -1$

1) $c = 0, c = -4$
2) $z = 5 + 2\sqrt{15}, z = 5 - 2\sqrt{15}$
3) $y = -3 + \sqrt{14}, z = -3 - \sqrt{14}$

1) $x = \frac{-5+\sqrt{17}}{2}, x = \frac{-5-\sqrt{17}}{2}$

2) $x = \frac{3+3\sqrt{5}}{2}, x = \frac{3-3\sqrt{5}}{2}$

SIMULTANEOUS EQUATIONS ANSWERS

1) $a = 3, b = -2$
2) $apple = 40p, banana = 10p$
3) $y = 5, x = -3$
4) $x = 4, y = -3 \; and \; x = 3, y = -4$
5) $x = 6, y = -1 \; and \; x = -3, y = -10$

ALGEBRAIC FRACTIONS ANSWERS

1) $u - 4$

2) $\frac{y-4}{y}$

3) $\frac{7v+4}{v(v+1)}$

4) $\frac{7-6f}{(2f-1)(2f+1)}$

5) $\frac{5}{(s+2)(s-3)}$

6) $\frac{9}{2(n-1)}$

7) $\frac{7(d+1)}{8}$

8) $\frac{4r(r+4)}{7}$

STRAIGHT LINES ANSWERS

1) $\frac{-3}{4}$
2) $(-2, 5)$
3) $6\sqrt{5}$

1) 3
2) $-\frac{3}{4}$
3) -3
4) 3

CIRCLE ANSWERS

1) $(0,0)$ $r = 5$
2) $x^2 + y^2 = 16$
3) $x^2 + y^2 = 2$